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Exotic baryons in two-dimensional QCD

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Abstract: Two-dimensional QCD has often been used as a laboratory for studying the full four-dimensional theory, providing, for example, an explicit realization of baryons as solitons. We review aspects of conventional baryons in two-dimensional QCD, including the classical and quantum contributions to their masses. We then discuss the spectrum of exotic baryons in two-dimensional QCD, commenting on the solitonic radius inferred from the excitation spectrum as well as the two-dimensional version of the Goldberger-Treiman relation relating meson couplings to current matrix elements. Our treatment of two-dimensional QCD, via the semi-classical quantization of collective coordinates, is consistent with chiral-soliton approaches to normal and exotic baryons in four dimensions, but is not able to resolve all the issues arising in four-dimensional models.

Keywords: Nonperturbative Effects, Field Theories in Lower Dimensions, Strong Coupling Expansion, QCD.

Contents

1. Introduction

Interest in baryons with 'exotic' quantum numbers - that cannot be composed of just three quarks - has been greatly stimulated recently by several reports of baryons whose composition must include at least four quarks and an antiquark — the so-called 'pentaquark' states. The first of these was the $\Theta^+(1530)$, with $\bar{s}udud$ quantum numbers [1], and others include the $\Xi^{--}(1860)$ [2] and the $\Theta_c(3100)$ [3]. The existence of even the Θ^+ cannot yet be regarded as confirmed, and the other states have each been seen in just one experiment. Nevertheless, reports of their existence have stimulated considerable new debate about baryon structure.

The existence of such 'exotic' baryons has long been predicted [4] by some chiral-soliton models [5], and a very specific prediction of the mass of the Θ^+ was made [6] years before its apparent confirmation. However, the coincidence between the predicted and observed masses may be just that, since there is still considerable uncertainty in the chiral-soliton estimates [7], and competing quark models have been proposed [8, 9] subsequent to the initial 'pentaquark' reports. Moreover, an alternative chiral-soliton analysis based on K-Nucleon scattering [10] finds no exotic resonances [11].

Two-dimensional QCD has long been considered a useful theoretical laboratory for studying non-perturbative strong-interaction issues such as confinement and the large-N_c expansion [12], deep-inelastic [13] and high-energy scattering [14], as well as baryon structure [15]. Clearly there are important differences between QCD in four and two dimensions, however. One example is provided by chiral symmetry and its breaking, on which we comment later in this paper. Nevertheless, two-dimensional QCD may provide interesting insights into the four-dimensional world.

The most convenient formulation for discussing baryons in two-dimension- -al QCD is in terms of bosonic meson fields. This bosonization is exact, and can be used to discuss baryons quantitatively as well as qualitatively, in the large-N_c limit. Bosonization methods and the non-exotic baryon spectrum and properties are reviewed in [15]. As we recall in the next section 2, baryons appear explicitly as solitons made out of the meson fields, with masses that are a factor $\mathcal{O}(N_c)$ larger. However, there are some important differences from QCD in four dimensions. Although the mass of the lowest meson vanishes as the quark mass $m_q \to 0$, it cannot be a Goldstone boson in two dimensions: instead, it decouples as $m_q \to 0$. Likewise, the baryon mass also vanishes in this limit. Its radius tends to a finite value in the large-N_c limit, which is of order $(m_q e_c)^{-1/2}$. As we discuss below, the baryon-meson coupling also vanishes in this limit, but in such a manner that the two-dimensional analogue of the Goldberger-Treiman relation is valid up to corrections that are $\mathcal{O}(m_q/e_c)$.

Since there are no spin degrees of freedom in two dimensions, the lowest-lying 'three-quark' baryons correspond to the purely symmetric Young tableau, and form a **10** representation of flavour SU(3), analogous to the Δ multiplet of four-dimensional QCD. The $\mathcal{O}(N_c)$ classical soliton mass can be calculated exactly, as can the first quantum correction, which is explicitly $\mathcal{O}(N_c^0)$. This provides an explicit formula for the two-dimensional analogue of the baryon moment of inertia, which is model-dependent in four-dimensional QCD.

As we discuss in section 3, the next excited state is a 'pentaquark' state with the quantum numbers of four quarks and an antiquark which, because of the quark wave-function symmetrization needed in two dimensions, forms a **35** representation of flavour SU(3). In four-dimensional chiral-soliton models, this multiplet is related to the $\overline{\mathbf{10}}$ that is thought to contain the Θ^+ and Ξ^{--} states, as well as a **27** representation, which have no analogues in two dimensions. The explicit mass formula for the **35** state in two dimensions is very similar to the general chiral-soliton formula in four dimensions [16], with the differences that there is no spin-dependent term $\propto J(J+1)$ and the single known soliton 'moment of inertia' appears.

One may continue to discuss higher-lying baryons which would require additional quark-antiquark pairs. The only allowed 'heptaquark' forms a totally symmetric **81** representation of SU(3), which is one of those appearing in four dimensions at this level of exoticity. In the large-N_c limit, the extra energy required to progress to the next level of exoticity: $qqq \to \bar{q}qqqq \to \bar{q}\bar{q}qqqqq \to \cdots$ is explicitly $\mathcal{O}(N_c^0)$, as previously predicted on the basis of four-dimensional chiral-soliton studies [16].[1]

2. Review of conventional baryons in two-dimensional QCD

2.1 The effective action

The natural form of the QCD_2 action is written in terms of gauge fields A_μ and fundamental

[1]However, as argued in [11], the corresponding $\mathcal{O}(N_c^0)$ splitting in the four-dimensional chiral-soliton picture is problematic.

quark fields Ψ:

$$S_F[\Psi, A_\mu] = \int d^2x \left\{ -\frac{1}{2e_c^2} \operatorname{Tr}(F_{\mu\nu}F^{\mu\nu}) - \bar{\Psi}^{ai}[(i\,\partial\!\!\!/ + A\!\!\!/\,)\Psi_i]_a + m_q \bar{\Psi}^{ai}\Psi_{ai} \right\}, \tag{2.1}$$

where e_c is the quark coupling to the gauge fields, which has the dimension of a mass in $1+1$-dimensional space-time, m_q is the common quark mass (we do not consider mass splittings in this paper), a is the color index and i the flavor index, and

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \tag{2.2}$$

is the gauge field strength.

In two dimensions, one may go over to a completely bosonic description, which is exact and particularly convenient for the discussion of the baryon spectrum. As discussed in [15], various bosonization schemes are available. Here we use the $U(N_F \times N_c)$ scheme, in which the QCD$_2$ action is rewritten as

$$\begin{aligned} S[u, A_+, A_-] = {} & S[u] - \frac{1}{2e_c'^2} \int d^2x \operatorname{Tr}(F_{\mu\nu}F^{\mu\nu}) + \\ & + \frac{i}{2\pi} \int d^2x \operatorname{Tr}(A_+ u\partial_- u^\dagger + A_- u^\dagger \partial_+ u) - \\ & - \frac{1}{2\pi} \int d^2x \operatorname{Tr}(A_+ u A_- u^\dagger - A_- A_+) + m'^2 N_{\tilde{m}} \int d^2x Tr(u + u^\dagger), \end{aligned} \tag{2.3}$$

where u is a bosonic $U(N_F \times N_c)$ matrix, $S[u]$ is the Wess-Zumino-Witten action:

$$S[u] = \frac{1}{8\pi} \int d^2x \operatorname{Tr}(\partial_\mu u \partial^\mu u^{-1}) + \frac{1}{12\pi} \int_B d^3y \varepsilon^{ijk} \operatorname{Tr}(u^{-1}\partial_i u)(u^{-1}\partial_j u)(u^{-1}\partial_k u), \tag{2.4}$$

$e_c' \equiv \sqrt{N_F} e_c$ and $m'^2 \equiv m_q c\tilde{m}$, where $\tilde{m}$ is the normal-ordering scale, which can be determined by convenience. The constant $c = \frac{1}{2}e^\gamma$, where γ is the Euler constant, so that $c \approx 0.891$.

Gauging the SU(N_c) subgroup and choosing the gauge $A_- = 0$, the QCD$_2$ action becomes

$$\begin{aligned} S[u, A_+] = {} & S[u] + \frac{1}{e_c'^2} \int d^2x Tr(\partial_- A_+)^2 + \\ & + \frac{i}{2\pi} \int d^2x \operatorname{Tr}(A_+ u \partial_- u^\dagger) + m'^2 N_{\tilde{m}} \int d^2x \operatorname{Tr}(u + u^\dagger), \end{aligned} \tag{2.5}$$

where $N_{\tilde{m}}$ denotes normal ordering with respect to the scale $\tilde{m}$.

In this gauge, the action is quadratic in the gauge potentials, which may therefore be integrated out. Taking also the strong-coupling limit, in which the gauge coupling e_c is much larger than the quark mass m_q, we can eliminate the color degrees of freedom entirely. Thus, we obtain an effective action expressed in terms of flavor degrees of freedom only:

$$S_{\text{eff}}[g] = N_c S[g] + m^2 N_m \int d^2x \operatorname{Tr}_F(g + g^\dagger), \tag{2.6}$$

where g is a matrix representing $U(N_F)$, and the effective mass scale m is

$$m = \left[N_c c m_q \left(\frac{e_c \sqrt{N_F}}{\sqrt{2\pi}} \right)^{\Delta_c} \right]^{\frac{1}{1+\Delta_c}}, \tag{2.7}$$

where the exponent

$$\Delta_c = \frac{N_c^2 - 1}{N_c(N_c + N_F)}. \tag{2.8}$$

We recognize (2.6) as an action of the type considered by Skyrme in QCD_4 to discuss baryonic solitons [5]. However, in QCD_2 it is the quark-mass term that plays the role of the stabilizing term, rather than the model-dependent higher-order terms used in QCD_4. Moreover, we emphasize that the above action is exact in QCD_2 in the strong-coupling (or small quark mass) limit.

In the large N_c limit, which we use below to justify the semi-classical approximation, the scale m tends to a constant times $\sqrt{N_c e_c m_q}$, where the constant is $0.56 N_F^{\frac{1}{4}}$, which takes the value 0.74 for three flavors.

Note that we first take the strong-coupling limit $e_c \gg m_q$, and then take N_c to be large. This is different from the 't Hooft limit [12], where $e_c^2 N_c$ is held fixed.

2.2 The classical soliton and its mass

In the spirit of the Skyrme model [5], we first examine classical soliton solutions of the bosonic action, which are heavy in the large-N_c limit. We examine later the quantum corrections, using the semi-classical approximation, and verify that they are small in the large-N_c limit, justifying *a posteriori* the assumption of a static, time-independent first step.

Without loss of generality, we may assume for the lowest-energy state a diagonal form of the matrix $g(x)$:

$$g(x) = \left(e^{-i\sqrt{\frac{4\pi}{N_c}}\varphi_1}, \ldots, e^{-i\sqrt{\frac{4\pi}{N_c}}\varphi_{N_F}} \right). \tag{2.9}$$

Using this Ansatz and redefining the constant term, the action density reduces to

$$\tilde{S}_d[g] = -\int dx \sum_{i=1}^{N_F} \left[\frac{1}{2} \left(\frac{d\varphi_i}{dx} \right)^2 - 2m^2 \left(\cos\sqrt{\frac{4\pi}{N_c}}\varphi_i - 1 \right) \right], \tag{2.10}$$

which is a sum of standard sine-Gordon actions. For each φ_i, the well-known solutions of the associated equations of motion are

$$\varphi_i(x) = \sqrt{\frac{4N_c}{\pi}} \operatorname{arctg} \left[e^{\sqrt{\frac{8\pi}{N_c}} m x} \right], \tag{2.11}$$

with the corresponding classical energy:

$$E_i = 4m\sqrt{\frac{2N_c}{\pi}} \qquad i = 1, \ldots, N_F. \tag{2.12}$$

Clearly the minimum energy configuration for this class of Ansatz is when only one φ_i is nonzero, for example

$$g_\circ(x) = \mathrm{Diag}\left(1, 1, \ldots, e^{-i\sqrt{\frac{4\pi}{N_c}}\varphi(x)}\right). \tag{2.13}$$

We interpret this state as the lowest-lying baryon [15].

In the large-N_c limit, the mass of this baryonic soliton is

$$M(\text{classical}) \approx 1.90 N_F^{1/4} \sqrt{e_c m_q} N_c\,, \tag{2.14}$$

In the particularly interesting case of three flavors, the coefficient becomes 2.50. We note that the mass is proportional to N_c, as we would expect. The proportionality to $\sqrt{e_c m_q}$ results from the form of the stabilizing term in (2.6), and is specific to two dimensions.

As in the four-dimensional case, this classical soliton solution has non-zero baryon number as well as Y charge, which is hypercharge normalized as a generator, so that the trace of Y^2 is 1/2:

$$Q_B^\circ = N_c \qquad Q_Y^\circ = -\sqrt{\frac{(N_F-1)}{2N_F}} N_c\,. \tag{2.15}$$

This observation follows from an explicit calculation of the charge densities of the soliton solutions, followed by their integration over space. We choose a normalization in which the quarks have baryon number $Q_B^\circ = 1$, so the soliton has baryon number N_c. This soliton thus represents a physical baryon [15].

2.3 Quantum corrections and allowed representations

We now calculate the quantum corrections. To this end, we allow the soliton to rotate in SU(N_F) space by a time-dependent amount $A(t)$:

$$g(x,t) = A(t) g_\circ(x) A^{-1}(t) \qquad A(t) \in U(N_F)\,, \tag{2.16}$$

We note that rotations by constant amounts are zero modes, and that charges of SU(N_F), other than the Y charge, now appear as results of such rotations. It is straightforward to derive the effective action for $A(t)$. The result is an extra contribution to the hamiltonian from these quantum oscillations, as well as a constraint on the allowed physical states, the latter coming from the Wess-Zumino term. After a lengthy calculation [15], we obtain for the mass

$$E = M(\text{classical})\left\{1 + \left(\frac{\pi}{2N_c}\right)^2 \left[C_2(R) - N_c^2 \frac{(N_F-1)}{2N_F}\right]\right\}, \tag{2.17}$$

where $M(classical)$ was given in (2.14) and $C_2(R)$ is the value of the quadratic Casimir for the flavor representation R of the baryon.

In order to determine the allowed quantum states, the first constraint is that of baryon number, which should be N_c. The second is that the representation R contains a state with the following value of the Y charge:

$$\bar{Q}_Y = \sqrt{\frac{1}{2(N_F-1)N_F}} N_c\,. \tag{2.18}$$

All other states will be generated by applying the appropriate $\mathrm{SU}(N_F)$ transformations to this one. Considering first states with only quarks and no antiquarks, the requirement that $Q_B = N_c$ implies that only representations described by Young tableaux with N_c boxes appear. The extra constraint that $Q_Y = \bar{Q}_Y$ implies that all N_c quarks are from $\mathrm{SU}(N_F-1)$, not involving the N_F'th quark flavor. These constraints are automatically obeyed in the totally symmetric representation of N_c boxes, which is, in fact, the only representation possible in two dimensions. This is because the states have to be constructed out of the components of one complex vector z as $\prod_{i=1}^{N_F} z_i^{n_i}$ with $\sum_i n_i = N_c$. In four dimensions there is also the possibility of some mixed representations that are not totally symmetric, such as the octet in the case of three colors and three flavors, as one can obtain the necessary total symmetry of the non-color part of the wave function by combining mixed-symmetry representations of flavor and spin. However, in QCD_2 for $N_c = 3, N_F = 3$ we get only the **10** of SU(3).

Since the form of the stabilizing term is fixed, as well as the leading quadratic term in the action and the Wess-Zumino term, the leading quantum correction to the mass is also known exactly. Combining this with the classical term, the mass of the **10** baryon becomes

$$M(\text{baryon}) = M(\text{classical}) \left[1 + \frac{\pi^2}{8} \frac{(N_F - 1)}{N_c} \right] , \tag{2.19}$$

where $M(classical)$ was given in (2.14). We note that the quantum correction is indeed suppressed by a factor of N_c as compared to the classical term, as expected in any number of dimensions. On the other hand, numerically the quantum correction ~ 0.82 for $N_c = 3, N_F = 3$, which is not very small.

2.4 Vibrational modes

The only static solutions of QCD_2 in the strong-coupling limit are the solitons we discussed above. As just discussed, their quantum corrections are obtained by time-dependent rotations in flavor space, which are suppressed by a factor of N_c compared to the classical contribution to the baryon mass.

In two dimensions, there are no degrees of freedom corresponding to spatial rotations, which is an important difference from the four-dimensional case. However, vibrational modes might in principle exist.

To look for higher excitations that appear as vibrational modes, one has first to look for time-dependent classical solutions. Taking again the strong coupling limit, and assuming a diagonal form for g, we find the following equation:

$$\partial_{tt}\varphi - \partial_{xx}\varphi + 2m^2 \sqrt{\frac{4\pi}{N_c}} \sin\left(\sqrt{\frac{4\pi}{N_c}} \varphi \right) = 0 \, , \tag{2.20}$$

where φ is one of the φ_i, in the notation of section 2.2. We see that, on dimensional grounds, φ is proportional to $\sqrt{N_c}$ times a function of $\frac{m.x}{\sqrt{N_c}}$ and $\frac{m.t}{\sqrt{N_c}}$.

It turns out that the only classical solution to this equation in the sector with baryon number $B = N_c$, is the lightest baryon we found above in the static case [17]. Thus, looking at the more general time-dependent case does not give any new single-baryon states. However, the time-dependent equation does give new meson states, as is well known in Sine-Gordon theory. The low-lying mesons have masses of order $\sqrt{m_q e_c}$, while the higher ones have masses of order N_c times that. We recall that the baryon masses are of the latter scale too, but even the highest meson is lighter than twice the lightest baryon mass.

In a confining theory like QCD$_2$, we would expect an infinite tower of mesons, rather than the finite set mentioned above. This restriction is due to our strong-coupling limit, where we keep only the lowest mesons whose masses $\sim \sqrt{m_q e_c}$, for fixed N_c. All the other mesons have masses $\sim e_c$, and so are infinitely heavier when e_c/m_q goes to infinity. In the limit studied by 't Hooft [12], in which N_c is large with $e_c^2 N_c$ fixed, such an infinite tower does indeed appear, and the mesons have squared masses

$$M_n^2 \sim \left(\frac{e_c^2 N_c}{\pi}\right)\pi^2 n \tag{2.21}$$

for large n.

3. Generalization to exotic baryons

3.1 The first exotic baryon

We now consider the case of the first exotic baryon $\mathcal{E}_1$, namely a state containing just one antiquark, which must also contain $N_c + 1$ quarks. In two dimensions, for the reasons discussed earlier, the allowed state must be totally symmetric in the quarks. For $N_c = 3, N_F = 3$, this state will be a $\mathbf{35}$ of flavor, to be compared with the $\overline{\mathbf{10}}$, $\mathbf{27}$ and $\mathbf{35}$ expected in four dimensions. The mass of this first exotic is easily found to be

$$M(\mathcal{E}_1) = M(\text{classical})\left[1 + \frac{\pi^2}{8}\frac{1}{N_c}\left(3 + N_F - \frac{6}{N_F}\right) + \frac{3\pi^2}{8}\frac{1}{N_c^2}\left(N_F - \frac{3}{N_F}\right)\right], \tag{3.1}$$

which is similar to the corresponding formula in [16]. In the interesting case $N_c = 3, N_F = 3$, the ratio of the mass of the first exotic to that of the lightest baryon is

$$\frac{M_{\mathbf{35}}}{M_{\mathbf{10}}} = \frac{1 + \frac{\pi^2}{4}}{1 + \frac{\pi^2}{12}}. \tag{3.2}$$

Numerically, this ratio is about 1.90. However, in this case the semi-classical approximation may not be a good approximation, as the quantum correction to the mass of the $\mathcal{E}_1$ state is larger than the classical term for $N_c = 3, N_F = 3$.

Here we are more concerned with the concepts and principles of the soliton model for baryons. On the other hand, we also note that the ratio of the experimental masses of the $\Theta^+(1530)$ and the nucleon is 1.63, so the ratio in the QCD$_2$ model is only ~ 17 % larger. However, this could be an accident, since the QCD$_2$ calculation does not take into account the fact that $m_s \neq m_{u,d}$ and the spin degree of freedom that is important in QCD$_4$.

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3.2 Higher-lying exotic baryons

We now consider higher-lying exotics containing p antiquarks and $N_c + p$ quarks, which we call $\mathcal{E}_p$ baryons. In two dimensions, according to the symmetry arguments already given above, the allowed states are totally symmetric in the quarks, and also totally symmetric in the antiquarks. In the standard case $N_c = 3, N_F = 3$, the only allowed $\mathcal{E}_2$ state is a **81** representation of flavor. For general N_c and p, considering the case $N_F = 3$ as an example, the mass of the $\mathcal{E}_p$ state is

$$M(\mathcal{E}_p) = M(\text{classical}) \left[1 + \frac{\pi^2}{4N_c^2}[N_c(p+1) + p(p+2)]\right], \tag{3.3}$$

corresponding to a correction that is considerably larger than unity. In this case, we would hesitate to advocate the semi-classical approximation for $N_c = 3$.

Nevertheless, we note that the spacing Δ between the $\mathcal{E}_{(p+1)}$ and $\mathcal{E}_p$ exotic states, for large N_c, behaves like

$$\Delta = \frac{\pi^2}{4}\frac{M(\text{classical})}{N_c}. \tag{3.4}$$

This amount becomes independent of N_c, and hence a constant additional mass, as the exoticity p is increased. We recall that meson masses are also $\mathcal{O}(N_c^0)$ for large N_C. This similarity supports the interpretation of this spacing as being analogous to the addition of a meson, which is indeed a quark-antiquark pair [16]. However, see the discussion in the Introduction regarding problems with this conclusion in four dimensions [11, 10].

Finally, we note that all the exotic baryons $\mathcal{E}_p$ with fixed exoticity $p \ll N_c$ have masses much smaller than those of the vibrational modes. Moreover, members of the $\mathcal{E}_p$ could mix with non-exotic **10** baryons only via SU(N_F)-breaking mass effects, and even this would be impossible for baryons with explicitly exotic combinations of the charge and hypercharge quantum numbers.

3.3 The size of the QCD$_2$ baryons

In four dimensions, the QCD soliton has two moments of inertia, which depend on the types of higher-order stabilizing terms used, and hence are model-dependent. As already noted, in QCD$_2$, the quantum correction to the mass depends on just one analogue of the moment of inertia. Its classical expression is $\int drr^2\rho(r)$, where $\rho(r)$ is the contribution to the classical soliton mass from a shell of radius r, $M(classical) = \int dr\rho(r)$. Hence, we can define an effective soliton radius by

$$\langle\langle r\rangle\rangle \equiv \sqrt{\langle r^2\rangle}, \qquad \text{where} \qquad \langle r^2\rangle = \frac{I}{M(\text{classical})}. \tag{3.5}$$

Comparing the quantum mass formula (3.3) with the corresponding formula in QCD$_4$ in the limit $N_c \gg p \gg 1$, we infer that

$$I = \frac{N_c^2}{\pi^2 M(\text{classical})}, \tag{3.6}$$

and hence that

$$\langle\langle r\rangle\rangle = \sqrt{\frac{I}{M(\text{classical})}} = \frac{N_c}{\pi M(\text{classical})} = \frac{1}{1.90\pi N_F^{1/4}\sqrt{e_c m_q}}\,. \tag{3.7}$$

We see explicitly that $\langle\langle r\rangle\rangle = \mathcal{O}(N_c^0)$, as expected also in four dimensions. The fact that $\langle\langle r\rangle\rangle$ depends on m_q reflects the special feature of the stabilizing term in QCD$_2$, and has no implications for four dimensions.

As a curiosity, note that, if we take $n_F = 3$ flavors, $e_c = 100\,\text{MeV}$ for the coupling and $m_q = 10\,\text{MeV}$ for the quark mass, we obtain an effective baryon radius $\approx 1/(248\,\text{Mev})$.

3.4 The Goldberger-Treiman relation in QCD$_2$

It is well known that a continuous symmetry cannot be broken spontaneously in two dimensions, and hence there can be no Goldstone bosons. On the other hand, the mass of the lowest-lying QCD$_2$ meson vanishes in the limit $m_q \to 0$, just like that of the four-dimensional pion. In two dimensions, however, the massless "pion" decouples, whilst in four dimensions it has has non-vanishing couplings in the limit $m_q \to 0$, which are proportional to axial-current matrix elements according to generalized Goldberger-Treiman relations:

$$g_{\pi NN} = \frac{2M_N}{f_\pi} g_A\,, \tag{3.8}$$

where (omitting irrelevant Lorentz factors) f_π is the coupling of the four-dimensional π meson to the axial current $\mathcal{A}$, and g_A is the matrix element of the axial current $\mathcal{A}$ in the nucleon N. In QCD$_2$, the flavor axial current $\mathcal{A}$ is directly related to the corresponding vector current $\mathcal{V}$:

$$\mathcal{A}_\mu = \epsilon_{\mu\nu}\mathcal{V}^\nu\,. \tag{3.9}$$

Hence, f_π and g_A are directly related to the flavor quantum numbers of the lowest-lying meson and baryon, respectively, and thus non-vanishing in the limit $m_q \to 0$. In QCD$_2$, the nucleon mass $\to 0$ as $m_q \to 0$, as we have already seen in (2.14), and so also do $g_{\pi NN}$ and the other π-baryon couplings. However, the ratio $\frac{g_{\pi NN}}{2M_N}$ is non-zero in this limit, being equal to $\frac{g_A}{f_\pi}$, as in the Goldberger-Treiman relation (3.8).

We firmly expect the corresponding generalized Goldberger-Treiman relations to hold for the couplings of light pseudoscalar mesons to baryonic solitons in QCD$_4$.

Finally, we note a corollary of the above remarks for the $\mathbf{35} - \mathbf{10}$ - meson coupling or, more generally, for meson couplings between baryons with differing degrees of exoticity. Such couplings would be related by generalized Goldberger-Treiman relations to matrix elements of axial currents $\mathcal{A}$ between baryons in different representations of SU(N_F), which are in turn related by (3.9) to off-diagonal matrix elements of vector currents $\mathcal{V}$. Since these must vanish, so also do the meson couplings between baryons with differing degrees of exoticity, at least in the large-N_c limit in which our approximation to QCD$_2$ is valid. This parallels the observation that the $K - N - \Theta^+$ vanishes in the large-N_c limit in QCD$_4$ [18].

How large may the $\mathbf{35} - \mathbf{10}$ - meson coupling be? Since it vanishes to leading order in N_c, we expect it to go to a constant in the large-N_c limit. A generic matrix element of the axial current between two baryon states has the dimensionality of a mass. In the

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large-coupling limit, the only relevant mass parameter is $\sqrt{e_c m_q}$, as we have seen above. Therefore, we expect the matrix element also to be proportional to $\sqrt{e_c m_q}$. This is consistent with its vanishing for zero quark mass, which we expect on the basis of the general arguments given above. There is no possibility of compensating for the large-N_c suppression by a different dependence on e_c.

Thus, the couplings between exotic baryons, normal baryons and chiral mesons have the same N_c^0 dependence expected in four dimensions. However, the two-dimensional case exhibits an extra m_q-dependent suppression that is absent in four dimensions: see the critiques of the collective-quantization approach to chiral solitons given in [10, 11].

4. Summary

QCD_2 provides a laboratory where the approximations made in deriving the chiral-soliton model [5] are explicit, and the corrections to the lowest-order calculations can be calculated exactly. In this paper, we have extended previous studies of baryons in QCD_2 [15] to include the analogues of the exotic baryons that have been predicted in four-dimensional QCD [6], which inspired experimental searches that have produced some positive reports [1−3]. There are differences specific to the structure of QCD_2, notably the absence of rotational degrees of freedom and also (in the strong-coupling regime considered here) of vibrational excitations. However, baryons in the two-dimensional model have many similarities to the expectations of various chiral-soliton approaches in QCD_4. We have exhibited explicitly the natures of the lowest-lying exotic baryons in two dimensions and calculated both their classical masses and the leading quantum corrections.

We consider that this analysis provides considerable support to the general four-dimensional chiral-soliton approach [5], without being able to render explicit the critiques in [10] and [11], because of the absence of rotational and vibrational excitations.

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